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Time Domain Properties of Non-maximal Length Sequences

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Abstract: *Linear Feedback Shift Register (LFSR) sequences find applications in spread spectrum communications, radars, system identification and cryptography. Among LFSR sequences, maximal length sequences are in variably used in these application due to their desirable properties. However maximal length sequences are available in limited number of lengths for a given number of shift register stages. On the other hand, the non-maximal length sequences can be generated in varieties of lengths. It appears that the NMLS were not investigated into, to the extent they deserve. The author conjectures that several applications other than communications and ranging might benefit from non-maximal length (NML) sequences. In this paper, NLM sequences are generated and their time domain properties are investigated in reference to the properties maximal length sequences. Simulations are carried out to generate NML sequences corresponding to polynomials of degrees from 6 to 20 and time domain properties such as Runs, One-zero Balance, Correlation, etc are investigated.*

Keywords: *Reducible polynomials, LFSR sequences, PN sequences, Correlation Analysis, Runs Property, Window Property, Shift and Add Property.*

I. Introduction

The Maximal Length Shift Register sequences are widely used in pulse compression radars [1], Random Number Generators [2], Spectrum Communication [3,4], Impulse Response Measurement [5,6,7], Transfer Function measurement [8, 9], Test Pattern Generation for circuit testing [10,11], Cryptography [12] and Programmable Sound Generators [13]. The Binary Linear Feedback Shift Register (BLFSR) Sequences are generated using a binary shift register with feedback connections and a multi-input modulo-2 (XOR) adder. The shift register with *n* stages connected with certain sets of feedback connections only give rise to sequences whose repetition periods are equal to $2^n - 1$. These sequences are called maximal length sequences (m-sequences), while other feedback connections generate sequences of repetition periods less than $2^n - 1$ (non-maximal length sequences). The m- sequences are invariably preferred over non-maximal sequences in most of the applications, due to their desirable properties. Accordingly, the non-maximal sequences were almost ignored and their properties were not explicitly discussed in the literature. The author conjectures that other non-communication like sound synthesis might benefit from non-maximal length sequences.

In this paper, the time domain properties of non-maximal length sequences corresponding to polynomials of degrees 6 to 20 are investigated with reference to the properties of m-sequences. The Runs property, the Balance property, the Correlation property, Window property, Shift property, Addition property and Shift-Add property are explored. The paper is organized as follows. In section II, the mathematical structure relating a LFSR sequence is discussed. In Section III the number of Non-maximal Length (NML) sequences available at each polynomial degree are computed. Section IV is dedicated to describe the simulations carried out to generate NML sequences and estimate their time domain properties. The result analysis is also carried out. Section V presents the conclusions and scope of future work.

II. Background Of Shift Register Sequences

A binary sequence $\boldsymbol{a} = a_0, a_1, a_2, \dots\dots$ can be generated by the recursion

$$a_k = h_1 a_{k-1} + h_2 a_{k-2} + \cdots + h_n a_{k-n} \quad (1)$$

where $h_0 = h_n = 1$ and other coefficients $h_i \in \{0,1\}$ and the initial condition vector $(a_{-n}, a_{-n+1}, \dots, a_{-2,} a_{-1})$ is non zero. The h_0 is the coefficient of the term a_k which is always unity. The binary vector $\boldsymbol{h} = (h_0, h_1, \dots, h_{n-1}, h_n)$ can be expressed as a polynomial *h(x)* as

$$h(x) = h_0 x^n + h_1 x^{n-1} + \dots + h_{n-1} x + h_n \quad (2)$$

which is called the generator polynomial of the sequence ***a***. Traditionally the binary vector $\boldsymbol{h} = (h_0, h_1, \dots, h_{n-1}, h_n)$ is represented either in binary or in octal notation

A 5-stage linear feedback shift register circuit with feedback specified by the polynomial $x^5 + x^3 + 1$ is shown in Fig.1. If the current output is taken from the k^{th} stage, then (k-1^{st}) is the previous stage, (k-2^{nd}) is the second previous stage and so on. The polynomial $x^5 + x^3 + 1$ can also be expressed as $1 + x^{-2} + x^{-5}$. Here a positive power means an advancement of a bit position, where as a negative power means a delay of the bit position. Thus both the polynomials represent the same shift register circuit. The former takes the reference stage at the right, while the latter considers the reference stage at the left. The binary vector corresponding to the polynomial x^5+x^3+1 is (101001) which is 51 in octal notation.

The sequence $\boldsymbol{a}$ generated by the LFSR circuit initially loaded with a non-zero binary vector in Fig.1 repeats every N bits. If the polynomial is a primitive irreducible, then the sequence repeats with a period of $N = 2^n - 1$, where n is the number of stages of the shift register which is same as the degree of the generator polynomial. This sequence is popularly known as maximal length sequence (MLS) or m-sequence [15]. The sequences generated with different initial states of the shift register (i.e. initial content of the shift register) are the same except for a cyclic shift. If the polynomial is reducible into factors, then $N < 2^n - 1$ and the sequences are called non-maximal length sequences (NMLS). In this paper the variable L is used to represent the length of NMLS and $L < N$.

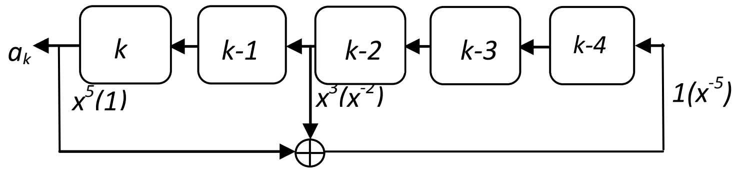

Figure1. A five stage linear feedback shift register with tap connections from x^5+x^3+1 or $1+x^{-2}+x^{-5}$.

The m-sequences [14,15,16] have a 2-level thumb-tack autocorrelation function and extremely low cross correlation function which make them unanimously preferred over m-sequences. The m-sequences are available in limited number of lengths i.e. $N = 2^n - 1$, whereas the NMLS are available in varieties of lengths.

III. Nonmaxmial Length Shift Register Sequences

For a given degree *n*, a polynomial must be irreducible if it has to generate either an m-sequence or a non-maximal length sequence while the *primitive irreducible* polynomials generate m-sequences, the *nonprimitive irreducible* polynomials generate non-maximal length sequences. In [17], the author computed all the possible periods of a NML sequence of a polynomial of degree n starting from 6 to 20. The polynomials used for the analysis are taken from [18] and the number of non-maximal length sequences available for each degree are found by using Möbius μ-function and Euler φ-functions as described in [14,17]. If $N = 2^n - 1$ has factors other than *1* and *N* itself, some of the polynomials corresponding to *n*, give rise to non-maximal length sequences of periods equal to the factors. These polynomials of non-maximal length sequences and their periods *L* for degrees 6 to 18 are given in Table 1. The maximal lengths *N* corresponding to *n=3, 5, 7, 13, 17* and *19* i.e. *7, 31, 127, 8191, 131071* and *524287* have no factors and hence can't generate any non-maximal length sequences. For degree 4 also, no *nonprimitive* polynomials or NML sequences exist.

Table1. Number of Non-maximal Length Sequences

Degree *n*	Period of NMLS	No. of NMLS	Total No of NMLS	Degree *n*	Period of NMLS	No. of NMLS	Total No of NMLS
6	21	2	3	15	4681	300	382
	9	1			1057	60	
8	85	8	14		217	12	
	51	4			151	10	
	17	2					
9	73	8	8				
10	341	30	39	16	21845	1024	2032
	93	6			13107	512	
	33	2			4369	256	
	11	1			3855	128	
					1285	64	
					771	32	
					257	16	
11	89	8	10	18	87381	2592	6756
	23	2			37449	1296	
12	1365	48	191		29127	864	
	819	36			13797	432	
	585	24			12483	432	
	455	24			9709	432	
	315	12			4599	144	
	273	12			4161	144	
	195	8			3591	108	
	117	6			1971	72	
	105	4			1533	48	
	91	6			1387	72	

	65	4			1197	36	
	45	2			657	24	
	39	2			513	18	
	35	2			399	12	
	13	1			219	8	
14	5461	378	405		189	6	
	381	18			171	6	
	129	6			133	6	
	43	3			57	2	
					27	1	
					19	1	

IV. Simulations And Results

Simulations are carried out to generate all non-maximal length sequences corresponding to polynomials of degree 6 to 18. The polynomial coefficients in octal form are converted to a binary vector $\boldsymbol{h} = (h_0, h_1, \dots, h_{n-1}, h_n)$ which gives the required feedback tap connections as described in Section II. All this conversion is done automatically using customized Matlab programs. Then the sequence is generated recursively using these feedback taps derived from $\boldsymbol{h}$. The recursive loop is continued to get a sequence $\boldsymbol{a}$ of *3.25N* binary digits, accounting for more than 4 - 8 cycles of non-maximal length sequence depending the actual period of the sequence. The value of 3.25 is not mandatory, but is used for getting multiple autocorrelation peaks to determine the sequence period unambiguously. The computed period $\hat{L}$, thus obtained for a sequence is cross checked with the period value obtained analytically by factoring the corresponding *N* value.

The simulation study included the generation of all 9840 non-maximal length sequences corresponding to degrees 6 to 18. The properties of the NML sequences are studied with reference to those of m-sequences and are analyzed. The results are populated in Table 2 for degrees 6 to 11 due to space constraints are analyzed . For each degree, all possible sequence periods are considered and the sequences (polynomials in octal form) grouped together. Each row in the Table 3 corresponds to the properties of an NML-sequence. The properties are discussed in what follows.

Property I – The Odd Periodicity

The period of a non-maximal length (NML) sequence is always odd as in case of an m-sequence.

Property II – The Shift Property

A cyclic shift of a non-maximal length (NML) sequence is also an NML sequence as in case of an m-sequence.

Property III – The window Property

When a sliding window of length *n* is moved along the NML sequence, the *n*-bit binary vector within the window occurs exactly once. All NML sequences of all degrees are found to satisfy this property. This is true for an m-sequence also.

Property IV – The Balance Property

Unlike an m-sequence, an NML sequence is not balanced i.e. number of 1s and 0s are not almost equal. In case of an m-sequence the number of 1s is more than the number of 0s by one. The actual number of 1s and 0s of NML sequences obtained through simulations are given in column 4 of Table 3.

Property V – The addition Property

The modulo-2 sum of two cyclic shifts of an NML sequence is another NML sequence. This is true for an m-sequence also.

Property VI – The Shift and Add Property

The modulo-2 sum of an NML-sequence and its cyclic shift is another NML-sequence. This is also true for m-sequences.

Property VII – The Runs Property

In case of an m-sequence, the number of runs of 1s or 0s follows a certain pattern. A run is string of consecutive 1's or 0's. The longer runs occur less frequently. In particular, the number of runs becomes half as the length of the run increases. This pattern is not followed by an NML-sequence. However, the longer runs occur rarely and vice verse as in case of m-sequences (columns 5 and 6 of Table 3).

Property VIII – Characteristic Phase

The characteristic phase of a sequence is defined as its cyclic shift which gives the same sequence when decimated by 2. All m-sequences have such a characteristic phase. The characteristic phase does not exist for NML sequences, except in very few cases such as sequences of either larger length or of higher degree. Whenever a characteristic phase does not exist for a sequence, it is shown as a '-' in the column 8 of Table 3. When it exists, the cyclic shift of the original sequence required to get the characteristic phase is given.

Property IX – Periodic Autocorrelation

The periodic autocorrelation function of an m-sequence is two-valued, like a thumb-tack i.e. single large peak at center and a small flat portion for all non-zero lags on both sides of the peak. However, the periodic autocorrelation function of an NML-sequence is multi-valued with a single peak at 0-th lag and 2 - 5 relatively smaller values for non-zero lags. The function is not flat but oscillates around 0. The actual values are given in column 7 of Table 2.

In the simulations study a total of 9840 sequences are generated and their repetition periods are measured using ACF function. The measured periods in simulations are compared to the theoretical periods obtained analytically using Möbius μ-function and Euler φ-functions as discussed in [14].

Table2. Ones-Zeros, Runs & Correlation Values of NML Sequences (degree: 6 to 11)

Degree	Period	Polynomial Octal	Num of 1s	0s	Runs of Ones 1,2,3,4,5,6,7,8,9,10	Runs of Ones 1,2,3,4,5,6,7,8,9,10	Correlation Values	Charact-eristic Phase
6	9	111	6	3	0,0,0,0,0,1	0,0,1,0,0,0	-3,1,5,9	-
	21	127	12	9	1,1,1,0,0,1	1,1,2,0,0,0	-3,5, 21	-
		165	12	9	1,1,1,0,0,1	1,1,2,0,0,0	-3,5, 21	-
8	17	727	12	5	0,2,0,0,0,0,0,2	2,0,1,0,0,0,0,0	-3,1,5,17	-
		471	10	7	2,0,0,0,0,0,0,1	1,0,2,0,0,0,0,0	-7,-3,1,5,17	-
	51	763	32	19	6,1,2,1,0,1,0,1	6,5,1,0,0,0,0,0	-13,3,51	43
		433	24	27	8,1,2,0,0,0,0,1	4,3,3,2,0,0,0,0	-13,3,51	-
		637	32	19	6,1,2,1,0,1,0,1	6,5,1,0,0,0,0,0	-13,3,51	1
		661	24	27	8,1,2,0,0,0,0,1	4,3,3,2,0,0,0,0	-13,3,51	-
	85	567	40	45	10,6,2,1,0,0,0,1	6,8,3,1,2,0,0,0	-11,5,85	-
		675	40	45	10,5,4,0,0,0,0,1	10,5,1,1,1,1,1,0	-11,5,85	-
		613	48	37	10,3,3,1,1,1,0,1	10,5,4,0,1,0,0,0	-11,5,85	21
		477	48	37	8,5,3,2,1,0,0,1	12,3,2,2,1,0,0,0	-11,5,85	77
		735	40	45	10,6,2,1,0,0,0,1	6,8,3,1,2,0,0,0	-11,5,85	-
		573	40	45	10,5,4,0,0,0,0,1	10,5,1,1,1,1,1,0	-11,5,85	-
		643	48	37	10,3,3,1,1,1,0,1	10,5,4,0,1,0,0,0	-11,5,85	57
		771	48	37	8,5,3,2,1,0,0,1	12,3,2,2,1,0,0,0	-11,5,85	1
9	73	1231	40	33	8,5,3,1,0,0,0,0,1	8,6,3,1,0,0,0,0,0	-7,1,17,73	-
		1027	40	33	12,3,3,1,0,0,0,0,1	14,2,2,1,1,0,0,0,0	-7,1,17,73	-
		1401	28	45	9,3,0,1,0,0,0,0,1	5,2,2,1,1,1,1,1,0	-7,1,17,73	0
		1511	40	33	5,3,3,0,1,1,0,0,1	5,3,4,1,0,1,0,0,0	-7,1,17,73	-
		1145	40	33	8,5,3,1,0,0,0,0,1	8,6,3,1,0,0,0,0,0	-7,1,17,73	-
		1641	40	33	12,3,3,1,0,0,0,0,1	14,2,2,1,1,0,0,0,0	-7,1,17,73	-
		1003	28	45	9,3,0,1,0,0,0,0,1	5,2,2,1,1,1,1,1,0	-7,1,17,73	65
		1113	40	33	5,3,3,0,1,1,0,0,1	5,3,4,1,0,1,0,0,0	-7,1,17,73	-
10	11	3777	10	1	0,0,0,0,0,0,0,0,0,1	1,0,0,0,0,0,0,0,0,0	7,11	1
	33	3043	20	13	6,2,0,0,0,0,0,0,0,1	6,2,1,0,0,0,0,0,0,0	-7,-3,1,5,9,33	-
		2251	20	13	2,4,0,0,0,0,0,0,0,1	3,2,2,0,0,0,0,0,0,0	-7,-3,1,5,9,33	-
	93	2065	48	45	4,4,4,1,2,0,0,0,0,1	4,4,4,1,2,0,1,0,0,0	-3,29,93	-
		3453	48	45	16,2,2,3,0,0,0,0,0,1	16,2,2,3,0,0,1,0,0,0	-3,29,93	-
		2413	48	45	12,8,2,1,0,0,0,0,0,1	12,8,2,1,0,0,1,0,0,0	-3,29,93	-
		2541	48	45	4,4,4,1,2,0,0,0,0,1	4,4,4,1,2,0,1,0,0,0	-3,29,93	-
		3205	48	45	16,2,2,3,0,0,0,0,0,1	16,2,2,3,0,0,1,0,0,0	-3,29,93	-
		3247	48	45	12,8,2,1,0,0,0,0,0,1	12,8,2,1,0,0,1,0,0,0	-3,29,93	-
	341	2017	160	181	48,26,7,2,3,1,0,0,0,1	40,26,11,5,2,2,2,0,0,0	-11,21,341	331
		2257	160	181	44,16,10,4,3,1,1,0,0,1	36,20,10,6,3,2,1,1,1,0	-11,21,341	89
		2653	176	165	36,22,11,4,2,2,1,1,0,1	36,22,11,6,3,1,1,0,0,0	-11,21,341	-
		3753	176	165	48,20,6,9,2,0,2,0,0,1	48,20,10,6,2,1,1,0,0,0	-11,21,341	-
		3573	176	165	44,22,12,6,1,1,1,0,0,1	52,14,10,9,1,0,2,0,0,0	-11,21,341	-
		2107	176	165	32,24,13,5,4,0,1,0,0,1	40,16,13,5,3,2,1,0,0,0	-11,21,341	-
		3061	176	165	48,18,12,4,2,2,0,1,0,1	48,18,14,3,4,0,1,0,0,0	-11,21,341	-
		2547	176	165	48,20,8,5,3,2,1,0,0,1	48,20,10,6,2,1,1,0,0,0	-11,21,341	-
		3121	176	165	36,22,9,7,1,3,0,1,0,1	36,22,11,7,1,2,1,0,0,0	-11,21,341	-
		2701	160	181	44,16,10,4,3,1,1,0,0,1	36,20,10,6,3,2,1,1,1,0	-11,21,341	155
		2437	160	181	44,16,10,4,3,1,1,0,0,1	36,20,10,6,3,2,1,1,1,0	-11,21,341	221
		2311	176	165	48,20,6,9,2,0,2,0,0,1	48,20,10,6,2,1,1,0,0,0	-11,21,341	-
		3607	160	181	40,24,6,6,0,2,0,1,0,1	40,16,10,4,4,2,2,1,1,0	-11,21,341	1
		2355	176	165	44,20,12,9,2,0,0,0,0,1	52,16,10,4,3,1,2,0,0,0	-11,21,341	-
		3315	176	165	48,16,14,5,2,1,0,1,0,1	48,20,10,6,2,1,1,0,0,0	-11,21,341	-

		3601	160 181	48,26,7,2,3,1,0,0,0,1	40,26,11,5,2,2,2,0,0,0	-11,21,341	1
		3651	160 181	44,16,10,4,3,1,1,0,0,1	36,20,10,6,3,2,1,1,1,0	-11,21,341	243
		3255	176 165	36,22,11,4,2,2,1,1,0,1	36,22,11,6,3,1,1,0,0,0	-11,21,341	-
		3277	176 165	48,20,6,9,2,0,2,0,0,1	48,20,10,6,2,1,1,0,0,0	-11,21,341	-
		3367	176 165	44,22,12,6,1,1,1,0,0,1	52,14,10,9,1,0,2,0,0,0	-11,21,341	-
		3421	176 165	32,24,13,5,4,0,1,0,0,1	40,16,13,5,3,2,1,0,0,0	-11,21,341	-
		2143	176 165	48,18,12,4,2,2,0,1,0,1	48,18,14,3,4,0,1,0,0,0	-11,21,341	-
		3465	176 165	48,20,8,5,3,2,1,0,0,1	48,20,10,6,2,1,1,0,0,0	-11,21,341	-
		2123	176 165	36,22,9,7,1,3,0,1,0,1	36,22,11,7,1,2,1,0,0,0	-11,21,341	-
		2035	160 181	44,16,10,4,3,1,1,0,0,1	36,20,10,6,3,2,1,1,1,0	-11,21,341	177
		3705	160 181	44,16,10,4,3,1,1,0,0,1	36,20,10,6,3,2,1,1,1,0	-11,21,341	111
		2231	176 165	48,20,6,9,2,0,2,0,0,1	48,20,10,6,2,1,1,0,0,0	-11,21,341	-
		3417	160 181	, 40,24,6,6,0,2,0,1,0,1	40,16,10,4,4,2,2,1,1,0	-11,21,341	331
		2671	176 165	44,20,12,9,2,0,0,0,0,1	52,16,10,4,3,1,2,0,0,0	-11,21,341	-
		2633	176 165	48,16,14,5,2,1,0,1,0,1	48,20,10,6,2,1,1,0,0,0	-11,21,341	-
11	23	5343	16 7	2 0 1 0 0 0 0 0 0 0 1	2 1 1 0 0 0 0 0 0 0 0	-1, 7, 23	-
		6165	16 7	2 0 1 0 0 0 0 0 0 0 1	2 1 1 0 0 0 0 0 0 0 0	-1, 7, 23	-
	89	4757	48 41	10,4,3,0,2,0,0,0,0,0,1	10,4,3,2,0,1,0,0,0,0,0	-23,-7, 9,89	-
		6777	56 33	8,6,1,1,2,0,0,1,0,0,1	12,4,3,1,0,0,0,0,0,0,0	-7, 9,89,	81
		7311	56 33	8,5,2,1,2,0,1,0,0,0,1	12,5,2,0,1,0,0,0,0,0,0	-7, 9,89	58
		4303	56 33	8,4,3,1,2,1,0,0,0,0,1	12,4,3,1,0,0,0,0,0,0,0	-7, 9,89	79
		7571	48 41	10,4,3,0,2,0,0,0,0,0,1	10,4,3,2,0,1,0,0,0,0,0	-23,-7, 9,89	-
		7773	56 33	8,6,1,1,2,0,0,1,0,0,1	12,4,3,1,0,0,0,0,0,0,0	-7, 9,89	87
		4467	56 33	8,5,2,1,2,0,1,0,0,0,1	12,5,2,0,1,0,0,0,0,0,0	-7, 9,89	21
		6061	56 33	8,4,3,1,2,1,0,0,0,0,1	12,4,3,1,0,0,0,0,0,0,0	-7, 9,89	0

Though all possible NML sequences for degrees up to 20 are simulated, the results are given for NML sequences for degrees 6 to 11 in Table 3, due to space constraints. Some sample NML sequences for selective degrees and periods are given in the Table 3. It may be observed that the shorter sequences are not fully random irrespective of the degree. For comparison purpose an m-sequence of degree 9 and polynomial 1665 (octal) is also given in the Table 3. This sequence is totally pseudo random unlike the NML sequences. This sequences satisfies all the Golomb postulates of pseudo-randomness [14]. All other sequences in Table 3 are NMLS and don't satisfy the Golomb postulates.

Two Sample NML sequences of degree 8 (length 85) and of degree 10 (length 341) and their two-sided autocorrelation functions are shown in Figures 1 and 2 respectively. Each binary sequence is shown as time waveform with [-1,1] levels and each bit is oversampled by 10 (or 8) samples for convenience of plotting. The autocorrelation function takes finite set of values [-11, 5, 85] in Figure 1 and [-11 21 341] in Figure 2. The values 85 and 341are the correlations at zero lag and other smaller values are the values at other non-zero lags. Another NML sequence of degree 11 and length 89 and its Autocorrelation Function is shown in Figure 3. Here each bit is oversampled by 16 samples since the sequence is shorter. This sequence takes 4 levels i.e. [-23 -9 7 89] . For comparison purpose, the m-sequence ***a6*** (L=511) given in Table 3 is plotted in Figure 4a and its autocorrelation function (ACF) is shown in figure 4b. The ACF takes only two levels -1 and 511 i.e. a thumb-tack function. The correlation value at all non-zero lags is -1 i.e. a very small value as expected.

V. Conclusions And Future Work

Time domain properties of the non-maximal length sequences generated by linear feedback shift registers are investigated. Simulations are carried out to generate a total of 9840 NML sequences corresponding to polynomials of degree 3 to 20. The period of each simulated sequence is computed. The measured periods and theoretically computed periods of all the simulated sequences are found to be exactly same. Selective NML sequences are also given. The properties of non-maximal length sequence are studied in reference to the properties of maximal length sequences. The author conjectures that several applications other than other communications might benefit from non-maximal length sequences. Some of the applications could be the sound synthesizer, clock dividers and the random number generators (RNG). Investigation by the author is in progress in this direction.

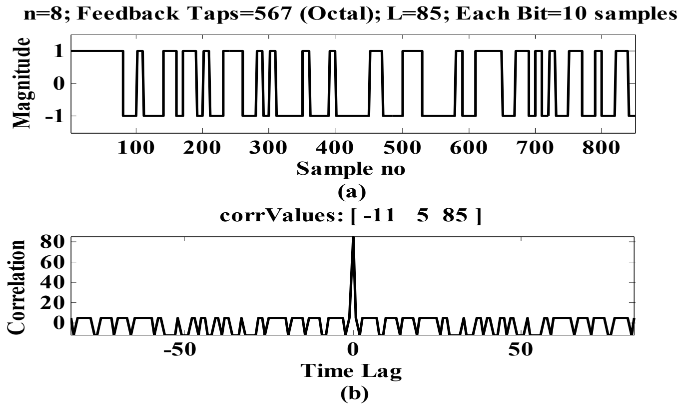

Figure1. A NML sequence of degree 8 and length 85 **(a).** Time waveform, bit 1 is given +1 and bit 0 is given -1. Each bit is oversampled by 10 samples for convenience of plotting **(b).** Autocorrelation function of the time waveform.

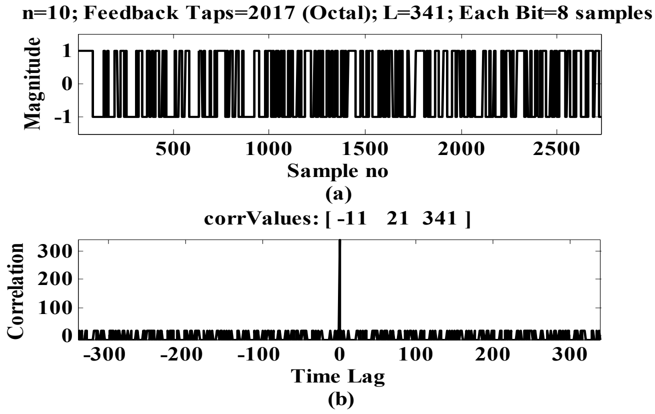

Figure2. A NML sequence of degree 10 and length 341 **(a).** Time waveform, bit 1 is given +1 and bit 0 is given -1. Each bit is oversampled by 8 samples for convenience of plotting **(b).** Autocorrelation function of the time waveform.

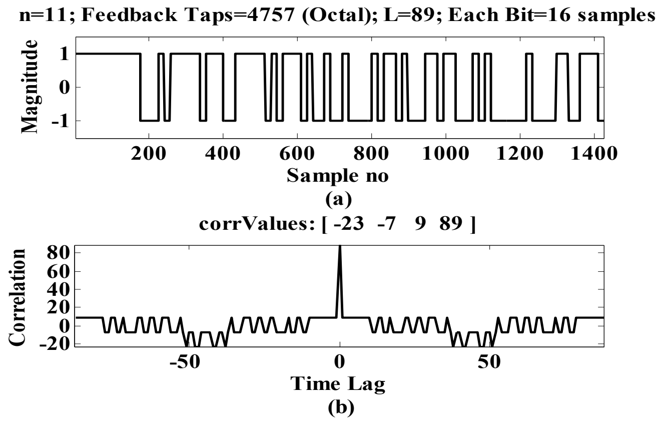

Figure3. A NML sequence of degree 11 and length 89 **(a).** Time waveform, bit 1 is given +1 and bit 0 is given -1. Each bit is oversampled by 16 samples for convenience of plotting **(b).** Autocorrelation function of the time waveform.

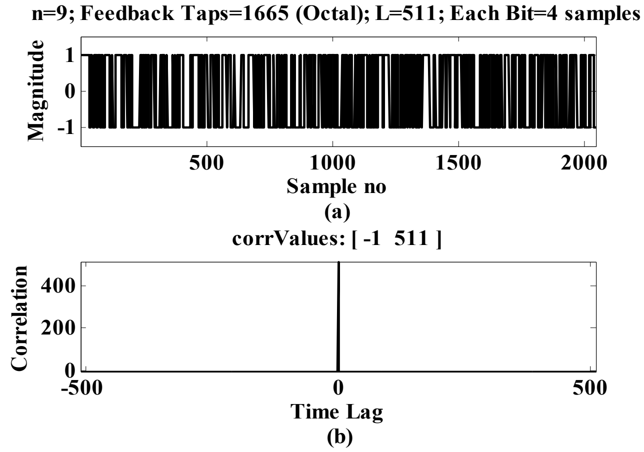

Figure4. The m-sequence ***a6*** of degree 9 (length 511**)** **(a).** Time waveform, bit 1 is given +1 and bit 0 is given -1. Each bit is oversampled by 4 samples **(b).** Autocorrelation function of the time waveform.

Matlab programs are also developed to display the results in ASCII string notation in compact form useful to populate the Tables 2 and 3.

Table3. Simulated sample Nonmaxmial Length Sequences &an m-Sequence (n=9 & L=511)

n=6 & L=21	**Polynomial1= 165 (Octal)** **a1=**111111011000100011100
n=8 & L=17	**Polynomial2= 471 (Octal)** **a2=**11111111000101000
n=8 & L=51	**Polynomial3= 661 (Octal)** **a3=**111111110100001000111011010001010010001001110010000
n=8 & L=85	**Polynomial4= 771 (Octal)** **a4=**1111111101111101001101110111000100001101101111011001010100001011101000001111001101000
n=9 & L=73	**Polynomial5= 1113 (Octal)** **a5=**1111111110001111100001101000111000110111000100101000000111111001110110010
n=9 & L=511	**Polynomial6= 1665 (Octal)** **a6=**11111111101100110111010011111001000111111001110010100000000110101011001110100001011001000001010010111100000000101111111010100111000111010110000111101110001000000111000001111101001001001111010001000110010011000000100100001000011101101101000111001100101011010100001101100011000100110110111001001010101111101111110001011010010100110101101100101101111111100001100001001110111011110101111011010111011000000100010100011010011001100011110001101111001101000001100111100101110101010100100010010110001010111000010101000101 1100 **(m-sequence)**
n=10 & L= 33	**Polynomial7= 2251 (Octal)** **a7=**111111111100011010011011001011000
n=10 & L=341	**Polynomial8= 2633 (Octal)** **a8=**11111111110010110000111101011010011101010110111011101100001010100010110110101110000111000101110001100101101010010010010111001101111111000101011111000111100101001111011000111011100111100010000001000001111101111010100011101011101010011001111111101110001001011001000011101001010000010001000010011010110110000000100101001010101100101010011100001 0

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